

Dynamic realization of the Unruh effect for a geodesic observer

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We study a dynamic version of the Unruh effect in a two dimensional collapse model forming a black hole. In this two-dimensional collapse model a scalar field coupled to the dilaton gravity, moving leftwards, collapses to form a black hole. There are two sets of asymptotic ($t \rightarrow \infty$) observers, around $x \rightarrow \infty$ and $x \rightarrow -\infty$. The observers at the right null infinity witness a thermal flux of radiation associated with time dependent geometry leading to a black hole formation and its subsequent Hawking evaporation, in an expected manner. We show that even the observers at left null infinity witness a thermal radiation, without experiencing any change of spacetime geometry all along their trajectories. They remain geodesic observers in a flat region of spacetime. Thus these observers measure a late time thermal radiation, with *exactly the same* temperature as measured by the observers at right null infinity, *despite moving geodesically in flat spacetime throughout their trajectories*. However such radiation, as usual in the case of Unruh effect, has zero flux, unlike the Hawking radiation seen by the observers at right null infinity. We highlight the conceptual similarity of this phenomenon with the standard Unruh effect in flat spacetime.

PACS numbers:

I. INTRODUCTION

Hawking radiation from a black hole [1–10] and the Unruh radiation [3, 6–8, 11, 12] in the Rindler frame have very similar mathematical properties. In the context of an eternal black hole, the Hartle-Hawking vacuum state of a quantum field leads to a thermal density matrix for static observers in the right wedge. This arises because the modes of the quantum field in the region inaccessible to the observer are traced out. Similarly, a uniformly accelerated observer in the right wedge of the flat spacetime will describe her observations using a thermal density matrix, obtained by tracing out the modes inaccessible to her (on the left wedge) when the field is in the global, inertial, vacuum state. Both situations are time-reversal invariant; while the relevant observers see an ambient thermal radiation, they do not associate a flux of particles with this radiation.

A somewhat different situation arises in the case of a black hole formed by collapsing matter, with the quantum state being the Unruh vacuum [In-vacuum] at very early times. In this case, at late times, observers far away from the collapsing body detect a flux of particles with a spectrum which is thermally populated. The energy carried away by the particles is ultimately obtained from the mass of the collapsing body and this leads to the concept of black hole evaporation. The mathematical description of this process takes into account; (i) the change in geometry due to the collapse process and (ii) the formation of event horizon leading to inaccessibility of a region from future asymptotic observers. With future applications in

mind, let us briefly recall the key concepts.

In standard 3+1 dimensional collapse, in which classical matter collapses to form a black hole, an apparent horizon is formed, which grows and — at a certain stage — an event horizon is formed thereby producing a black hole region. The last null ray originating from the past null infinity \mathcal{J}^- at the null co-ordinate $u = d$, in the double null co-ordinate system [1, 3, 6–8], reaching future null infinity \mathcal{J}^+ defines the location of the event horizon. An asymptotic observer on \mathcal{J}^+ has no causal connection with events inside the event horizon. Further, whole of \mathcal{J}^+ derives its complete causal support (e.g., the thick orange line in Fig. 1) from only a *part of* \mathcal{J}^- , which lies prior to the ray forming event horizon. A wave mode

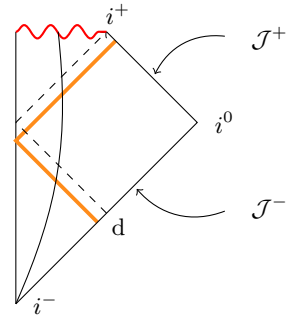


FIG. 1: (color online) Penrose Diagram For Schwarzschild collapse

which originates from \mathcal{J}^- and reaches \mathcal{J}^+ also experiences a change in background geometry in the process. In order to obtain the Bogoliubov coefficients between

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the asymptotic observers¹ on \mathcal{J}^- and \mathcal{J}^+ , we need to re-express a set of complete mode functions v_ω suited to observers on \mathcal{J}^+ in terms of complete set of mode functions u_ω , defined equivalently on \mathcal{J}^- . When we trace back the out-going modes v_ω on \mathcal{J}^+ through the center, (i.e., through $r = 0$ with r being the Schwarzschild radial co-ordinate) onto \mathcal{J}^- , we see that the mode functions suited to \mathcal{J}^+ have support only on the portion of mode functions u_ω on \mathcal{J}^- .

Therefore the Bogoliubov coefficients are obtained by taking inner products of u_ω and v_ω on a portion of \mathcal{J}^- , i.e., below the line $u = d$ [1, 3, 7, 8]. (This has an effect similar to that of partial tracing out of modes in the case of eternal black hole.) Furthermore, we now also have to take into account the change of geometry experienced by the mode function u_ω when we trace it back to the past null infinity. However, if we are interested only in the late time behavior of u_ω , we can use ray-optics approximation [1, 3, 7, 8] for tracing back the mode functions. *Thus, the late time behavior of the out going asymptotic future modes is essentially controlled by the tracing over of portion of \mathcal{J}^- , rather than the change in the geometry.* This is the mathematical reason why the dynamics of the collapse is irrelevant to obtain the temperature and one obtains the same result as in the case of an eternal black hole and Hartle-Hawking vacuum state. The exact Bogoliubov coefficients, obtained by going beyond the ray optics, will, of course, be sensitive to the geometry change as well. Further, it is the loss of time reversal invariance during the collapse which leads to a non-zero flux of energy, which is absent in the case of an eternal black hole and Hartle-Hawking vacuum state.

It turns out that a combination of these effects arise in the case CGHS black hole in 1+1 dimension [5, 13]. The collapse of quantum matter leads to the standard black hole evaporation scenario with observers at future right asymptotic $\mathcal{J}_R^+(t \rightarrow \infty, x \rightarrow +\infty)$ detecting a flux of thermal radiation, which is well-known in the literature. This situation is mathematically identical to what happens in the case 1+3 spherically symmetric collapse. But it turns out that there is another effect in the same spacetime which is very similar to Unruh effect: Observers at future left asymptotic $\mathcal{J}_L^+(t \rightarrow \infty, x \rightarrow -\infty)$ detect a thermal spectrum (at the same temperature as seen by observers at \mathcal{J}_R^+) but without any associated flux! This result is surprising because these are geodesic observers in a flat region of the spacetime who see no change in geometry. The effect arises due to the necessity of tracing over part of the modes for describing the physics of the observers at \mathcal{J}_L^+ and the mathematics closely parallels standard Unruh effect. But in this case, the necessity to trace over modes arises due to the dynamics of collapse,

which takes place in a different region of the spacetime though. This effect (which, as far as we know, has been missed in the CGHS literature) constitutes a dynamics realization of Unruh effect.

II. A PICTURE BOOK REPRESENTATION

In order to explain the concepts involved in this, rather peculiar result, we will first provide a picture-book description of how the result arises, which should demystify it.

We start with the Minkowski spacetime, for which the Penrose diagram corresponds to Fig. 2. The full spacetime is bounded by four null lines depicting future and past left/right null infinities, viz \mathcal{J}_R^+ , \mathcal{J}_R^- , \mathcal{J}_L^+ and \mathcal{J}_L^- . In this spacetime any inertial (geodesic) observers will start from past timelike infinity i^- at $t = -\infty$ and would reach future timelike infinity i^+ at $t = \infty$. Two such geodesic observers moving leftwards and rightwards respectively, are shown by dashed curves in Fig. 2. In addition, there can also be some accelerated observers. An important set of such accelerated observers is the eternally accelerating Rindler observer. The trajectory of the Rindler observer starts on \mathcal{J}_L^- and accelerates along a hyperbolic path to reach \mathcal{J}_L^+ (shown in the thick green curve in Fig. 2). Let us consider the causal support of the trajectories of the Rindler observer vis-à-vis the inertial observer. The inertial observer has causal access to the full spacetime, whereas the spacetime region accessed by the Rindler observer is only a part of the full Minkowski spacetime. Thus the vacuum state for the inertial observer (who can access the full spacetime) would be different from that of the Rindler observer (for whom only a part of the spacetime is allowed) leading to non zero Bogoliubov coefficients. Alternatively, the observations carried out by the Rindler observer can be described using a density matrix obtained by tracing out modes inaccessible to her. If the field is in the inertial vacuum state, the resulting density matrix will be thermal. This property will hold whenever an observer has a causal access of only a part of the spacetime and the reduced density matrix she uses will be thermal. As an illustration let us consider Fig. 3, where three observers are shown. The dashed observer is an inertial observer and, as in the earlier figure, has access to the full spacetime; the dotted trajectory is that of the standard, eternally accelerating Rindler observer; the third trajectory (represented by thick magenta line) represents an observer who originally started as inertial but then changed her mind and accelerates uniformly to end up on \mathcal{J}_L^+ just like the Rindler observer. This observer also has causal connection only to the past domain of dependence of \mathcal{J}_L^+ , which being only a portion of the full manifold, will lead to thermal density matrix.

Let us now consider a hypothetical scenario in which some portion of Minkowski spacetime becomes dynamically inaccessible, as shown in Fig. 4 by the dashed tri-

¹ We can think of observers moving with large speeds and reaching future timelike infinity. These observers, in an approximating limit, may be mimicked through late time observers on \mathcal{J}^+ .

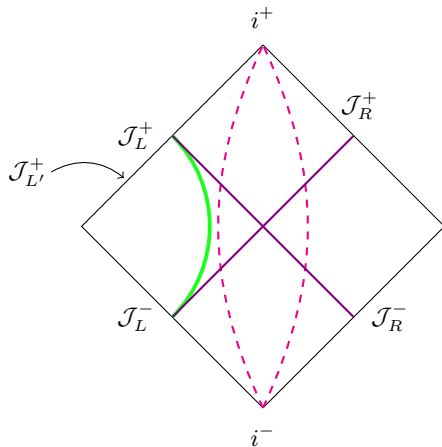


FIG. 2: (color online) Rindler trajectory in Minkowski spacetime

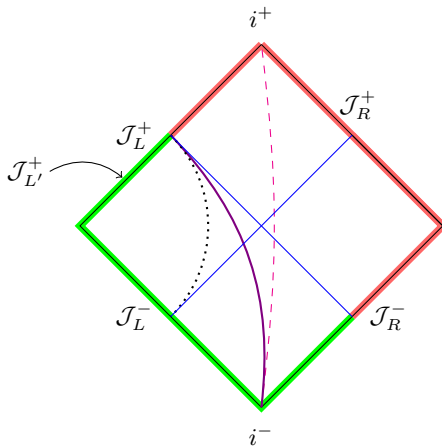


FIG. 3: (color online) Non-geodesic Observer in Minkowski spacetime

angular region. (Right now this is the Penrose diagram of some hypothetical spacetime; we will soon see how it actually arises in the CGHS case.) Further, we assume this truncation of spacetime is such that it requires left-moving geodesic observers to terminate on i_L^+ instead of i^+ as in Fig. 3. Since these observers derive their causal support only from the past of J_L^+ , they are compelled to trace over a portion of field configuration on J_R^- and hence they will end up using a density matrix which is thermal. Whenever *any* observer, *geodesic or not*, who derives her causal support from a subset of the full spacetime, the global vacuum state will appear to be a thermal state.

The geodesic part of the above statement might appear perplexing. One may wonder how a geodesic observer can ever experience such a nontrivial effect usually associated with accelerated observers. The role of acceleration is only to make part of the spacetime region inaccessible;

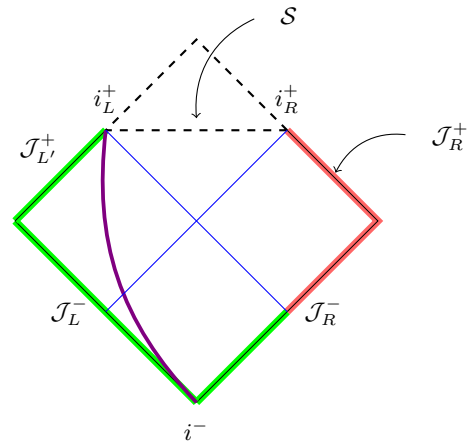


FIG. 4: (color online) Observers in a hypothetical spacetime

if we can achieve this by some other means, we will still have the same result. The situation as depicted in Fig. 4 appears unphysical as presented — because we have artificially removed part of the spacetime —, but we will later discuss a situation in which a portion of spacetime is indeed dynamically denied to a *geodesic observer*, very much like in the spirit of the observer in Fig. 4 (shown in thick magenta curve).

But before we do that, we will consider another example in the next section which might make all these less surprising. This will involve the collapse of a null shell forming a black hole (see Fig. 5). In the case, a time-like geodesic observer at $r = 0$, will remain entirely in the flat spacetime until being eaten up by the singularity and receives causal signals only from a part of J^- (see Fig. 5 below). This study will provide us useful insights towards the constructs to be used in the later sections of the paper.

III. A NULL SHELL COLLAPSE

For a closer look at the above mentioned features, we will first consider a null shell collapse forming a Schwarzschild hole [3, 7, 14–16] (see Fig. 5). In this case, the singularity gets originated from the co-ordinate $u = u_i$, the co-ordinate point of introduction of the null-shell. (This collapse model will be relevant for comparison with the null shell collapse in 1 + 1 dimensional dilatonic gravity, forming a black hole, which is discussed later on.) The singularity starts forming from the co-ordinate $u = u_i$, while the event horizon is located at $r = 2M$, with M being the mass of the shell. The co-ordinate $r = 2M$ can be reflected back through $r = 0$ on to J^- at $u = d$. As discussed previously, the out-going modes v_ω on J^+ derive their causal support completely from the portion $u < d$ on J^- . Therefore the Bogoliubov coefficients of mode transformation, should be evaluated through the portion of mode functions u_ω in the regime

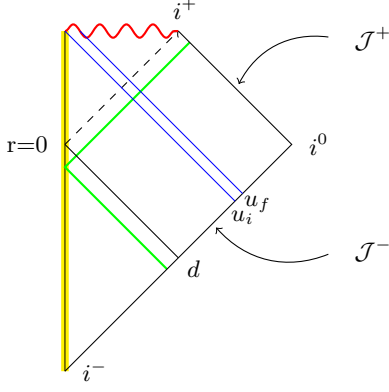


FIG. 5: (color online) Penrose Diagram For Schwarzschild null shell collapse

$u < d$. Further, all the null rays emanating from \mathcal{J}^- region $u < u_i$ also experience a change of geometry, i.e., they start moving in flat spacetime inwards, get reflected at the origin ($r = 0$) and then encounter the collapsing shell to feel the geometry changed into a Schwarzschild one.

This can be more clearly seen in an isotropic co-ordinate system (see Fig. 6) which uses a Cartesian x coordinate with $-\infty < x < \infty$ instead of the usual radial coordinate r with $0 < r < \infty$. The relevant Penrose diagram is shown in Fig. 6. In these coordinates, (see

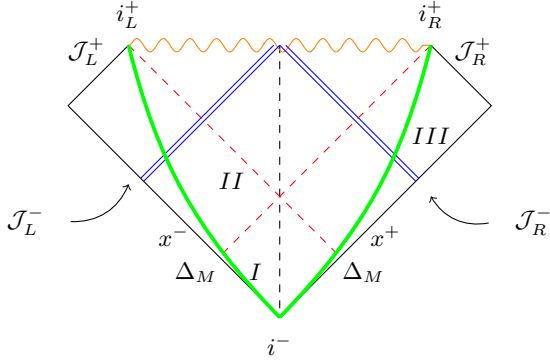


FIG. 6: (color online) Schwarzschild in isotropic co-ordinates

Fig. 6) there will be two past null infinities, namely left (\mathcal{J}_L^-) and right (\mathcal{J}_R^-), as well as two future null infinities \mathcal{J}_L^+ and \mathcal{J}_R^+ . The location of event horizon in these co-ordinates is marked to (say) Δ_M , which corresponds to $r = 2M$ in Schwarzschild co-ordinates. It is usual to define the modes as left moving or right moving in these co-ordinates. A right moving mode originates from \mathcal{J}_L^-

and ends up on \mathcal{J}_R^+ and vice versa for the left-moving modes. Due to spherical symmetry, consideration of any set of past and future asymptotic observer pairs will be equivalent to any other. A time-like observer (thick green curves in Fig. 6), similarly starting from past time like infinity and escaping the black hole region, either end up on i_L^+ or on i_R^+ depending upon whether the observer moves leftwards or rightwards.

Let us consider a set of modes moving rightwards. Any null ray originating from \mathcal{J}_L^- and ending up on \mathcal{J}_R^+ will experience a change of geometry after it crosses the collapsing null shell. For observers on \mathcal{J}_R^+ , the isotropic co-ordinate $|x^-| = \Delta_M$ marks the location of event horizon. Therefore, the modes reaching \mathcal{J}_R^+ derive their causal support from the region I in Fig. 6. No event in the region II is connected to \mathcal{J}_R^+ . An exactly similar picture is there for the left-moving modes.

In the standard black hole analysis, the out-going mode functions on \mathcal{J}_R^+ are given as

$$v_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega u}; \quad u \in (-\infty, \infty). \quad (1)$$

These mode functions provide an orthonormal basis across complete \mathcal{J}_R^+ . Similarly the right-moving modes on \mathcal{J}_L^- are spanned by

$$u_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega v}; \quad v \in (-\infty, \infty). \quad (2)$$

The Bogoliubov transformation coefficients between these modes can be evaluated by taking the covariant inner products on \mathcal{J}_L^- . However, in order to do that we need to express mode functions v_ω on \mathcal{J}_L^- in terms of u_ω . For that purpose we need to track the out-going modes at \mathcal{J}_R^+ all the way down to \mathcal{J}_L^- . In principle, this a formidable job, since the exact form of modes in the whole spacetime is complicated at best, if obtainable in closed form. However, appealing to ray-optics approximation [1, 7] for modes very close to the horizon, we in a sense, avoid this issue. This approximation gives us the expression of modes v_ω close to $|x^-| = \Delta_M$, i.e., $u = d$ in terms of u_ω . We obtain the Bogoliubov coefficients readily as

$$\begin{aligned} \alpha_{\omega\omega'} &= -2i \int_{-\infty}^0 dx^- u_\omega \partial_- u_{\omega'}^*, \\ \beta_{\omega\omega'} &= 2i \int_{-\infty}^0 dx^- u_\omega \partial_- u_{\omega'}. \end{aligned} \quad (3)$$

However, as we discussed this integration has to be truncated to within the region $u < d$, which gives it a Rindler kind of appearance, making it [1, 3, 7]

$$\begin{aligned}\alpha_{\Omega\omega} &= \frac{1}{2\pi\kappa} \sqrt{\frac{\Omega}{\omega}} \exp\left[\frac{\pi\Omega}{2\kappa}\right] \exp[i(\Omega - \omega)d] \exp\left[\frac{i\Omega}{\kappa} \log \frac{\omega}{C}\right] \Gamma\left[-\frac{i\Omega}{\kappa}\right], \\ \beta_{\Omega\omega} &= -\frac{1}{2\pi\kappa} \sqrt{\frac{\Omega}{\omega}} \exp\left[-\frac{\pi\Omega}{2\kappa}\right] \exp[i(\Omega + \omega)d] \exp\left[\frac{i\Omega}{\kappa} \log \frac{\omega}{C}\right] \Gamma\left[-\frac{i\Omega}{\kappa}\right].\end{aligned}\quad (4)$$

where $C = C_1 C_2$ is a product of affine parameters for in-going (C_1) and out-going (C_2) null geodesics. From the rather general nature of the analysis we expect this result to give the Bogoliubov coefficients for the timelike geodesic observer stationary at $r = 0$ if we replace d by u_i . Such an observer is engulfed by a *true* singularity when the shell collapse to $r = 0$, it results in geodesic incompleteness, something we will come back to, in the last section.

In the limit $M \rightarrow 0$, the portion being traced over becomes small. Further the geometry change as well as the formation of the singularity does not occur, making \mathcal{J}^+ a Cauchy surface, which is not the case when $M \neq 0$. Since the effect of tracing over is intimately tied with the effect of geometry change, it is difficult to account for these effects individually in a general case in 1+3 dimension. Nevertheless, one can argue that the tracing over of

modes alone leads to Unruh effect with zero flux, whereas the geometry change makes the thermal radiation more ‘real’ with a non-zero flux [17–20]. The non vanishing of flux can be associated with moving of the geometry away from being a flat one. We will see below that these two effects can nicely be segregated in a 1 + 1 dimensional collapse model in dilaton gravity.

IV. 1+1 DIMENSIONAL DILATONIC BLACK HOLE

The CGHS black hole [5, 13] is a 1+1 dimensional gravitational collapse model of a dilatonic field ϕ interacting with gravity in the presence of cosmological constant λ and matter fields f_i , described by the action,

$$\mathcal{A} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[e^{-2\phi} (R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right]. \quad (5)$$

Since all two dimensional space-times are conformally flat the metric ansatz will involve a single unknown function, the conformal factor ρ , which is written in double null coordinates as,

$$ds^2 = -e^{2\rho} dx^+ dx^-, \text{ with,} \quad (6)$$

$$x^+ \in (0, \infty), \text{ and } x^- \in (-\infty, 0).$$

For the matter fields, the classical solutions are those in which, $f_i = f_{i+}(x^+) + f_{i-}(x^-)$. Given some particular matter fields one can obtain corresponding solutions for ϕ and ρ respectively from the equations of motion. A simple static solution corresponds to $e^{-2\rho} = e^{-2\phi} = (M/\lambda) - \lambda^2 x^+ x^-$, representing a black hole of mass M , with a line element

$$ds^2 = -\frac{dx^+ dx^-}{\frac{M}{\lambda} - \lambda^2 x^+ x^-}. \quad (7)$$

In absence of the mass, $M = 0$ and we obtain *linear dilaton vacuum* line element from Eq.(7).

We consider, for the collapsing scenario, a simplistic case where only one scalar field f is present. The matter

moving leftwards collapses to form a black hole [13, 21]. If the matter distribution starts at x_i^+ and extends up to x_f^+ , then the line element corresponding to this matter configuration turns out to be,

$$ds^2 = -\frac{dx^+ dx^-}{\frac{M(x^+)}{\lambda} - \lambda^2 x^+ x^- - P^+(x^+) x^+}, \quad (8)$$

where the functions $M(x^+)$ and $P^+(x^+)$ correspond to the integrals,

$$M(x^+) = \int_{x_i^+}^{x^+} dy^+ y^+ T_{++}(y^+), \quad (9)$$

$$P^+(x^+) = \int_{x_i^+}^{x^+} dy^+ T_{++}(y^+). \quad (10)$$

The region outside x_f^+ is a black hole of mass $M \equiv M(x_f^+)$ [5]. There is a curvature singularity at $e^{-\rho} = 0$. The singularity hides behind an event horizon (which is located at $x^- = -P^+/\lambda^2$) for future null observers receiving the out-moving radiation. The location of the

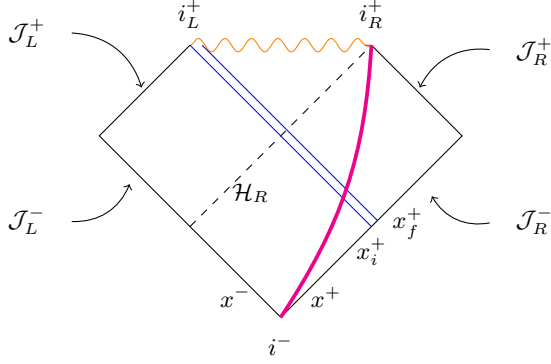


FIG. 7: (color online) Penrose Diagram for a CGHS black hole

event horizon can be obtained starting from the location of the apparent horizon, see Fig. 7. This can be obtained using $\partial_+ A \leq 0$, where A stands for the transverse area of the horizon in a 3 + 1 dimensional setting. Borrowing this idea to 1 + 1 dimension, this equality would lead to the location of the event horizon at $x^- = -P^+/\lambda^2$, in the out region, i.e., after $x^+ > x_f^+$.

Thermodynamics as well as Hawking evaporation of such black hole solutions have been extensively studied [5, 21–25]. We introduce a co-ordinate set z^\pm suited for the in-region \mathcal{J}_L^-

$$\pm \lambda x^\pm = e^{\pm \lambda z^\pm}, \quad (11)$$

which maps the entire \mathcal{J}_L^- into $z^- \in (-\infty, \infty)$. We also introduce another co-ordinate system suited for \mathcal{J}_R^+ as $\sigma_{\text{out}}^\pm \in (-\infty, \infty)$ where the transformation between (z^+, z^-) and $(\sigma_{\text{out}}^+, \sigma_{\text{out}}^-)$ is given by:

$$z^+ = \sigma_{\text{out}}^+; \quad z^- = -\frac{1}{\lambda} \ln \left[e^{-\lambda \sigma_{\text{out}}^-} + \frac{P^+}{\lambda} \right]. \quad (12)$$

The horizon located at $x^- = -P^+/\lambda^2$, will get mapped to $z^- = z_i^- = -\frac{1}{\lambda} \log(P^+/\lambda)$ in these co-ordinates. ‘In-’ state modes are defined on the asymptotically flat region

\mathcal{J}_L^- moving towards \mathcal{J}_R^+ and the convenient basis modes defined correspond to,

$$u_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega z^-}, \quad (13)$$

where $\omega > 0$. The ‘out’ region corresponds to \mathcal{J}_R^+ which receives the state from \mathcal{J}_L^- after the black hole has formed. The basis modes in the out region at \mathcal{J}_R^+ are

$$v_\omega(\sigma_{\text{out}}^-) = \frac{1}{\sqrt{2\omega}} e^{-i\omega \sigma_{\text{out}}^-}; \quad (14)$$

$$v_\omega(z^-) = \frac{1}{\sqrt{2\omega}} e^{-i\omega \sigma_{\text{out}}^-(z^-)} \Theta(z_i^- - z^-), \quad (15)$$

where Θ is the usual step function marking the fact that the out modes are supported by states on \mathcal{J}_L^- between the region $(-\infty, z_i^-)$ only. These mode functions provide a complete orthonormal basis on \mathcal{J}_R^+ in terms of σ_{out}^- , however. Again the field can be specified fully on \mathcal{J}_L^- or jointly on \mathcal{J}_R^+ and the event horizon \mathcal{H}_R . Since the mode functions at \mathcal{H}_R correspond to part of the field falling into the singularity and such interior modes cannot be observed by observers at \mathcal{J}_R^+ , they need to be traced over. Thus, the precise form of mode decomposition on \mathcal{H}_R does not affect physical results for \mathcal{J}_R^+ . Therefore, we can expand the dilaton field in different mode basis as,

$$f = \int_0^\infty d\omega [a_\omega u_\omega + a_\omega^\dagger u_\omega^*], \quad (\text{in}) \quad (16)$$

$$= \int_0^\infty d\omega [b_\omega v_\omega + b_\omega^\dagger v_\omega^* + \hat{b}_\omega \hat{v}_\omega + \hat{b}_\omega^\dagger \hat{v}_\omega^*], \quad (\text{out}) \quad (17)$$

where a_ω^\dagger corresponds to creation operator appropriate for the ‘in’ region. Similarly b_ω^\dagger and \hat{b}_ω^\dagger stand for the creation operators for the ‘out’ region and the black hole interior region respectively. The inner product between v_Ω and u_ω^* corresponds to,

$$\begin{aligned} \alpha_{\Omega\omega} &= -\frac{i}{\pi} \int_{-\infty}^{z_i^-} dz^- v_\Omega \partial_- u_\omega^* = \frac{1}{2\pi} \sqrt{\frac{\omega}{\Omega}} \int_{-\infty}^{z_i^-} dz^- \exp \left[\frac{i\Omega}{\lambda} \ln \left\{ \left(e^{-\lambda z^-} - \frac{P^+}{\lambda} \right) \right\} + i\omega z^- \right] \\ &= \frac{1}{2\pi\lambda} \sqrt{\frac{\omega}{\Omega}} \left(\frac{P^+}{\lambda} \right)^{i(\Omega-\omega)/\lambda} B \left(-\frac{i\Omega}{\lambda} + \frac{i\omega}{\lambda}, 1 + \frac{i\Omega}{\lambda} \right), \end{aligned} \quad (18)$$

while the inner product between v_Ω and u_ω gives

$$\begin{aligned} \beta_{\Omega\omega} &= \frac{i}{\pi} \int_{-\infty}^{z_i^-} dz^- v_\Omega \partial_- u_\omega = \frac{1}{2\pi} \sqrt{\frac{\omega}{\Omega}} \int_{-\infty}^{z_i^-} dz^- \exp \left[\frac{i\Omega}{\lambda} \ln \left\{ \left(e^{-\lambda z^-} - \frac{P^+}{\lambda} \right) \right\} - i\omega z^- \right] \\ &= \frac{1}{2\pi\lambda} \sqrt{\frac{\omega}{\Omega}} \left(\frac{P^+}{\lambda} \right)^{i(\Omega+\omega)/\lambda} B \left(-\frac{i\Omega}{\lambda} - \frac{i\omega}{\lambda}, 1 + \frac{i\Omega}{\lambda} \right), \end{aligned} \quad (19)$$

with $B(x, y)$ being the Beta function. Therefore, we can verify that the late time right moving observers (such a observer is depicted by the thick magenta curve in Fig. 7) do obtain a thermal spectrum [5] with a temperature $1/\lambda$ from

$$N_\Omega = \int_\omega |\beta_{\Omega\omega}|^2. \quad (20)$$

Interestingly, all references to the matter content which formed the black hole, appears only in the phase (see Eq.(18) and Eq.(19)), through P^+ and gets wiped out when we take the modulus. If we follow a null ray originating from \mathcal{J}_L^- or a timelike trajectory and moving rightwards, it suffers a change of geometry once it crosses the collapsing null shell, as in the case for Schwarzschild hole formation. Therefore, the experiences of these observers are inclusive of both tracing over and the geometry change. Hence, as associated with a black hole formation, there is an associated flux of radiation as measured by the observers attached to such trajectories [5, 21]. However, this model being asymmetric under left-right exchange, provides an opportunity of studying the effect of tracing over separately, if we consider the left-moving modes, which we do next.

V. DYNAMICAL UNRUH EFFECT IN CGHS BLACK HOLE SPACETIME

The CGHS model is not symmetric under left-right exchange, and hence the experiences of null ray originating from \mathcal{J}_R^- or a timelike trajectory moving leftwards, will be different from what we discussed above. Such trajectories do not suffer any change in geometry in their course, hence, as we will see, the only effect a late time observer (the dotted curve in Fig. 8) on \mathcal{J}_L^+ finds, is rooted only in the tracing over a part of Cauchy surface \mathcal{J}_R^- . Therefore, these observers will also obtain a thermal expectation value in Eq.(20) as we will see, but there will be no associated flux for these thermal spectrum. The spacetime

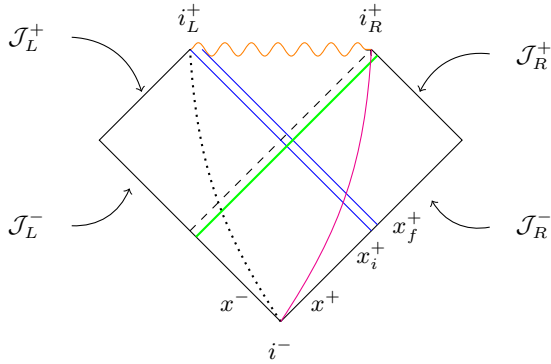


FIG. 8: (color online) Left moving modes in a CGHS black hole

metric in the region $x^+ < x_i^+$ is given as flat, given by Eq.(8) with $M, P^+ \rightarrow 0$, which on using the co-ordinates

$$x^+ = -\frac{1}{\lambda y^+}; \quad x^- = -\frac{1}{\lambda y^-}, \quad (21)$$

can be written as

$$ds^2 = -\frac{dy^+ dy^-}{-\lambda^2 y^+ y^-}. \quad (22)$$

The singularity curve originates from the co-ordinate of the point of matter introduction, i.e., from x_i^+ which is marked through Eq.(21) as $y_i^+ = -1/\lambda x_i^+$. Under another set of co-ordinate transformations, the metric on \mathcal{J}_R^- can also be brought into flat form. On \mathcal{J}_R^- we adopt

$$e^{-\lambda\chi^+} = -y^+, \quad e^{\lambda\chi^-} = y^-, \quad (23)$$

such that the metric becomes

$$ds^2 = -d\chi^+ d\chi^-, \quad (24)$$

with $\chi^\pm \in (-\infty, \infty)$. Therefore the complete set of left-moving mode functions corresponding to the field $f_+(\chi^+)$ can be written in these co-ordinates as

$$u_\omega^+(\chi^+) = \frac{1}{\sqrt{2\omega}} e^{-i\omega\chi^+}. \quad (25)$$

Whereas on \mathcal{J}_L^+ we adopt to

$$e^{-\lambda\tilde{\chi}^+} = -(y^+ - y_i^+), \quad e^{\lambda\tilde{\chi}^-} = y^-, \quad (26)$$

such that these new co-ordinates range in $\tilde{\chi}^\pm \in (-\infty, \infty)$ in the region $x^+ < x_i^+$. The metric in these new co-ordinates becomes

$$ds^2 = -\frac{d\tilde{\chi}^+ d\tilde{\chi}^-}{1 - y_i^+ e^{\lambda\tilde{\chi}^+}}. \quad (27)$$

The metric Eq.(27) is in a conformally flat form in these co-ordinates, hence the complete set of left-moving modes corresponding to $f_+(\tilde{\chi}^+)$ are again given as

$$v_\omega^+(\tilde{\chi}^+) = \frac{1}{\sqrt{2\omega}} e^{-i\omega\tilde{\chi}^+}, \quad (28)$$

since a minimally coupled scalar field is conformally invariant in 1 + 1 dimensions. Clearly, on \mathcal{J}_R^- , v_ω^+ has support only in the region $x^+ < x_i^+$. The point x_i^+ is mapped to $\tilde{\chi}^+ \equiv \tilde{\chi}_i^+ = -\frac{1}{\lambda} \log(-y_i^+)$. Therefore, the Bogoliubov transformation coefficients between these two set of observers can be obtained exactly as in Eq.(18), Eq.(19) but with the replacement $|y_i^+| \leftrightarrow P^+/\lambda$ as

$$\begin{aligned}
\alpha_{\Omega\omega} &= -\frac{i}{\pi} \int_{-\infty}^{x_i^+} d\chi^+ v_{\Omega} \partial_- u_{\omega}^* = \frac{1}{2\pi} \sqrt{\frac{\omega}{\Omega}} \int_{-\infty}^{x_i^+} d\chi^+ \exp \left[\frac{i\Omega}{\lambda} \ln \left\{ \left(e^{-\lambda\chi^+} - |y_i^+| \right) \right\} + i\omega\chi^+ \right] \\
&= \frac{1}{2\pi\lambda} \sqrt{\frac{\omega}{\Omega}} |y_i^+|^{\frac{i(\Omega-\omega)}{\lambda}} B \left(-\frac{i\Omega}{\lambda} + \frac{i\omega}{\lambda}, 1 + \frac{i\Omega}{\lambda} \right),
\end{aligned} \tag{29}$$

while

$$\begin{aligned}
\beta_{\Omega\omega} &= \frac{i}{\pi} \int_{-\infty}^{x_i^+} d\chi^+ v_{\Omega} \partial_+ u_{\omega} = \frac{1}{2\pi} \sqrt{\frac{\omega}{\Omega}} \int_{-\infty}^{x_i^+} d\chi^+ \exp \left[\frac{i\Omega}{\lambda} \ln \left\{ \left(e^{-\lambda\chi^+} - |y_i^+| \right) \right\} - i\omega\chi^+ \right] \\
&= \frac{1}{2\pi\lambda} \sqrt{\frac{\omega}{\Omega}} |y_i^+|^{\frac{i(\Omega+\omega)}{\lambda}} B \left(-\frac{i\Omega}{\lambda} - \frac{i\omega}{\lambda}, 1 + \frac{i\Omega}{\lambda} \right).
\end{aligned} \tag{30}$$

This form of Bogoliubov coefficients results from the tracing over of a portion of \mathcal{J}_R^- , which is causally disconnected from \mathcal{J}_L^+ . The horizon for \mathcal{J}_L^+ is given as $y^+ = y_i^+$, which also marks the point of singularity. More importantly, if one is interested in the late time response, i.e. the observers reaching i_L^+ , we need to take the high frequency behavior ($\omega/\lambda \gg 1$) of the Bogoliubov coefficients which are exactly the same as for the Rindler observer [6].

This effect can be more clearly understood using non-availability of Cauchy surfaces for future asymptotic observers. Let us consider a left moving timelike or null trajectory. Due to conformal flatness, a complete set of orthonormal mode functions on any surface orthogonal to them, can always be obtained as plane waves under a proper co-ordinatization (which extends as $(-\infty, \infty)$) of that surface. If we consider an orthogonal null surface for the left moving modes, before the formation of singularity (the red surface labeled ‘2’ in Fig. 9), the Bogoliubov transformation coefficients between that surface and \mathcal{J}_R^- will be trivial, i.e., $(\alpha_{\Omega\omega} = \delta(\Omega - \omega), \beta_{\Omega\omega} = 0)$. However,

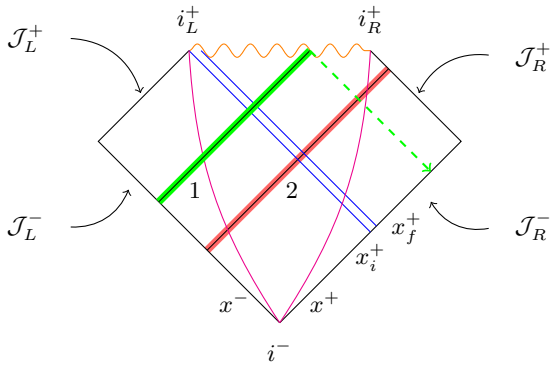


FIG. 9: (color online) Unavailability of Cauchy surface for left-moving modes

once the singularity forms, any null surface relating a point on \mathcal{J}_L^- to the singularity (e.g., green surface la-

beled ‘1’ in Fig. 9)), fails to causally connect with the entire spacetime. Also, any timelike observer has a domain of dependence, which is not the full spacetime. A portion of \mathcal{J}_R^- (right to the dashed green line) is causally denied to the past of such surfaces. Therefore, the Bogoliubov coefficients assume a non-trivial form, such as in Eq.(18), Eq.(19) or Eq.(29), Eq.(30), with a phase factor capturing the information about the region of the traced over modes. This is dynamically similar to a Rindler trajectory in a Minkowski spacetime, see Fig. 2, where an accelerated observer (thick green curve) reaches the future null infinity rather than future timelike infinity. For such observers too, the domain of dependence is only a part of the full spacetime. There is a horizon masking a portion of spacetime and hence the Bogoliubov transformation coefficients between \mathcal{J}_R^- and \mathcal{J}_L^+ (rather than the full \mathcal{J}_L^+) is non-trivial [3, 6, 8] and the accelerated observers obtain a thermal spectrum for a vacuum state defined on \mathcal{J}_R^- . However, there is no flux associated with this spectrum as the stress tensor remains identically vanishing in the flat spacetime.

Similarly in the CGHS model, the region of spacetime $y^+ > y_i^+$ is dynamically made inaccessible to any left-moving time-like or null trajectories, which also is the case with such trajectories in the null collapse forming a Schwarzschild black hole Fig. 6. However, in contrast to the Schwarzschild case, such observers in the CGHS model, do not see any change of geometry hence they do not associate any mass to the “black hole region” as seen by them. The spacetime, they move in, throughout, is flat and such observers do not receive any flux of radiation, as the vacuum expectation value of the stress energy tensor which was vanishing on \mathcal{J}_R^- , stays put on zero, in the region $y^+ < y_i^+$. As we discussed previously Eq.(20), the right moving observers tuned to right moving modes, witness a thermal radiation flux at a temperature $1/\lambda$ which is independent of the matter content of the forming black hole and depends only on the other dimensionful parameter in the theory.

Similarly, the observers at \mathcal{J}_L^+ coupled to left-moving modes, observe a similar kind of Bogoliubov coefficients

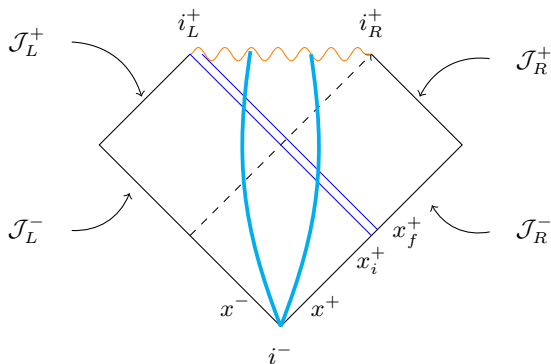


FIG. 10: (color online) Freely falling observers in CGHS black hole spacetime

as their right-moving counterparts Eq.(18), Eq.(19) but with the parameter exchange $|y_i^+| \leftrightarrow P^+/\lambda$ which mark the corresponding event horizons for such observers and appear as overall phases in the transformation coefficients. Hence, we see that such form of Bogoliubov coefficients Eq.(29), Eq.(30) are entirely due to tracing over of modes which lie in the causally denied region of spacetime, to \mathcal{I}_L^+ and not due to any geometry change. One can check that if the fraction of tracing over vanishes, which is marked by $|y_i^+| \rightarrow 0$, the Bogoliubov coefficients assume a trivial form, i.e., $\alpha_{\Omega\omega} \rightarrow \delta(\Omega - \omega)$ while $\beta_{\Omega\omega} \rightarrow 0$. Similarly, the Bogoliubov coefficients for the right-moving observers Eq.(18), Eq.(19), assume a trivial form in the limit $P^+ \rightarrow 0$, marking the vanishing of event horizon, as well as, the amount of change of geometry suffered by such observers. Hence, for such observers effects of tracing over is indistinguishable from geometry change. Both these effects vanish simultaneously in that limit. Therefore, we see that the vacuum response for both left-moving or right-moving observers is indistinguishable from each other. Late time radiation for such observers on \mathcal{J}_L^+ or on \mathcal{J}_R^+ for vacuum state (of a test field) is thermal. However, unlike the left-moving observers, the right moving observers associate a flux also with the radiation and hence the black hole region shrinks as a result of the evaporation. While left-moving observers do not associate any mass to the region beyond their horizon, the location of their horizon does not shrink and the “black hole region” does not evaporate for them.

All the geodesic observers moving rightwards or remaining stationary at any finite value (with the exception of stationary observer at left infinity) end up in singularity and in this process have to undergo a geometry change (thick cyan curves in Fig. 10), so they witness a combined effect and hence a flux of radiation, as in the Schwarzschild scenario [15, 16, 26]. Whereas the left-moving timelike observers end up at i_L^+ , do not see any geometry change but encounter flux-free thermal radiation due to a dynamical emergence of a horizon, which

traces out information of a section of field configuration, exactly in the spirit of the Rindler observer, as discussed previously.

Finally, we will comment on the back-reaction due to black hole evaporation and its implications for our result. To compute any kind of back-reaction in the spacetime in Einstein gravity, one needs to use an equation of the form $G_{ab} = \langle T_{ab} \rangle$. *Neither side of this equation can be handled without additional assumptions and, obviously, the result will depend on the additional assumptions!* The left-hand-side identically vanishes in $D = 2$ because Einstein tensor identically vanishes in two dimensions. The right hand side is: (i) divergent and needs to be regularized and (ii) depends on the quantum state which is chosen. The best we can do, therefore, is to give prescriptions to define both sides and the results will obviously *depend on these prescriptions*. We discuss our set of choices while describing the pertinent issues in two-dimensional gravity below.

To get a non-zero left hand side, one postulates some non-Einsteinian form of the gravity action (like the CGHS action in Eq.(5)) and vary the metric to get the equations of motion. Obviously, this is not G_{ab} (which, of course, is zero) but can possibly act as a proxy for the same. We stress the fact that we are trying to model some gravitational features by some suitable dilatonic action and one cannot ignore the implicit ad-hocness in the procedure. For example, in such an approach, one also often takes the the normal ordered, non-covariant, expectation value of the stress energy tensor as the classical source. However, in a two-dimensional spacetime, the expectation value of the stress energy tensor comes with a conformal anomaly term [5] as well. The anomaly term can then act as an extra source of stress energy and may lead to an evolution different from the classical case, if not accounted for properly. A prescription used in the literature is to add some extra terms (e. g., Polyakov action, RST action terms) to the standard CGHS action corresponding to the conformal anomaly terms and obtain these *modified* equations of motion, as a result of the variational principle. But this new action will correspond to a different physical system, though one which can also be thought of as a proxy for gravity. But in this prescription, flat spacetime will not be a solution to the vacuum state for the semiclassical equations due to the anomaly, which we consider somewhat unphysical. There is no unique way of handling this issue because, as we stressed before, $G_{ab} = 0$ in $D = 2$ and the left hand side which we work with to mimic $G_{ab} = \langle T_{ab} \rangle$ depends crucially on the model we use, with the hope that it can mimic aspects of $D = 4$ gravity. We have chosen simple, physically well-motivated choices to do this as we describe next.

To analyze the right hand side, we need a scheme for defining a c -number stress tensor from the quantum operator and the vacuum state has to be motivated from some specific (geometric) considerations. We have *defined* the vacuum state by a natural assumption: viz.

that the geometry must remain flat when sourced by such a (vacuum) state. This is important because as we discussed, in two-dimensional spacetime, the expectation value of the stress energy tensor has a conformal anomaly term which can violate this criterion in general, if this term is not accounted for in the source terms in the RHS. However, there exists a family of such states [27] which take care of the conformal anomaly terms and ensure that the geometry remains flat when the state is a vacuum from this family. Therefore the results discussed in this work, use stability of the solution under these class of states. A more detailed analysis of conformal anomaly modified evaporating black holes can be found in [5]. Our scheme to deal with such terms will be reported in details elsewhere [28]. At last, we discuss the response of an Unruh-DeWitt detector carried by such a geodesic observer.

VI. UNRUH-DEWITT DETECTOR RESPONSE

To confirm the fact operationally, that an observer who entirely stays in the left block of spacetime and has access to only a portion of full spacetime, truly finds the vacuum state of a field defined/supported in the full spacetime as an excited state on her Hilbert space defined in the portion of spacetime, we turn to analysis of response of an Unruh-DeWitt detector carried by a left-moving inertial observer.

Such an observer describes the spacetime metric as in Eq.(24). We construct a co-ordinate system $\{x \equiv (T, X)\}$

$$\begin{aligned}\chi^+ &= T + X \\ \chi^- &= T - X\end{aligned}\quad (31)$$

to write Eq.(24) as

$$ds^2 = -dT^2 + dX^2 \quad (32)$$

s.t. the Wightman function, corresponding to the vacuum defined in full spacetime, in 1+1 dimension assumes the form

$$\mathcal{W}(x, x') = -\frac{1}{2} \log |x - x'|^2 = -\frac{1}{2} \log \Delta\chi^+ \Delta\chi^-. \quad (33)$$

A left moving inertial observer is given by the trajectory

$$\frac{dT}{d\tau} = \gamma; \quad \frac{dX}{d\tau} = -v\gamma,$$

where v is the inertial velocity and γ is the corresponding Lorentz factor. If the spacetime were flat throughout, i.e., no matter shell had collapsed, the inertial observer, carrying the detector, would have exhausted the full range of proper time $\tau \in (-\infty, \infty)$. The appearance of singularity due to collapse of the matter shell leads to geodesic incompleteness and hence regulate the proper time interval of the geodesic observer to $(-\infty, \tau_f)$, with τ_f marking the location i_L^+ (see the left curve in Fig. 9).

In 1+1 dimension, the response of a monopole detector coupled to a (test) field $\Phi(x)$ is ambiguous [29]. Therefore, we work with derivative field detector with interaction Hamiltonian

$$\mathcal{H}_{\text{int}} = c\mu(\tau) \frac{d}{d\tau} \Phi(x(\tau)), \quad (34)$$

where c is a coupling constant, $\mu(\tau)$ is detector's monopole moment operator and τ is the proper time detected by observer carrying the detector. The detector response function [29], for a two level detector, with level separation $\hbar\omega$, will be given for this case as

$$\mathcal{F}^{(1)}(\omega) = \int_{-\infty}^{\tau_f} d\tau \int_{-\infty}^{\tau_f} d\tau' e^{-i\omega(\tau-\tau')} \partial_\tau \partial_{\tau'} \mathcal{W}(\tau, \tau'). \quad (35)$$

In flat portion of the spacetime, on any trajectory, if the test field $\Phi(x)$ is in the vacuum state,

$$\partial_\tau \partial_{\tau'} \mathcal{W}(\tau, \tau') = \frac{1}{(\tau - \tau')^2}, \quad (36)$$

becomes solely a function of $\Delta\tau = \tau - \tau'$. Had the spacetime been geodesically complete, $\tau_f \rightarrow \infty$ and the detector would not have clicked. However, due to collapsing matter field and the subsequent removal of a portion of spacetime, the spacetime becomes geodesically incomplete. In such a case, the detector response will be analogous to a finite time inertial detector [30], which indeed registers clicks. However, notice that here we have not introduced any artificial switching function, but it is the inextensibility of the geodesic, which leads to the non-zero response function. In the case there is no black hole collapse, the detector will fall silent.

We can now transform to co-ordinates $\Delta\tau$ and a mean proper time $\bar{\tau} = (\tau + \tau')/2$ to re-write Eq.(35) as

$$\mathcal{F}^{(1)}(\omega) = \int_{-\infty}^{\tau_f} d\bar{\tau} \int_{-(2\tau_f-2\bar{\tau})}^{(2\tau_f-2\bar{\tau})} \frac{d\Delta\tau}{(\Delta\tau)^2} e^{-i\omega\Delta\tau}. \quad (37)$$

Therefore, the detection rate becomes time dependent as fit for a detector moving through an excited state [31], i.e. at different points on the trajectory the detector will click differently. Hence, we show that the detector also conforms to the notion that the state as measured by the left moving geodesic observer is non-vacuum. With a bit of exercise [30], one can evaluate the clicking rate of the detector at any general point of time as well. The detector clicks with a non-zero rate and the left-moving observer concludes that there are particles in her patch of the spacetime.

It is important to note that this detector is constructed in a manner such that it will not register any clicking, if the quantum field resides in the vacuum state of the observer carrying the detector along (e.g. Inertial observer in Minkowski spacetime in regular 3+1 dimensional gravity). Therefore, the global vacuum defined on \mathcal{J}_R^- fails to be a vacuum state for left-moving geodesic observers. Further, the detector does not click in a steady fashion

at a thermal detection rate, but this is due to the fact that the detector response depends hugely on the trajectory unlike the Bogoliubov transformation. For instance, it is easy to demonstrate that in Fig. 3 the non-geodesic observer (thick magenta curve) and the Rindler observer (dotted trajectory) obtain the same Bogoliubov coefficients as they derive the casual support from the same patch, yet Unruh-DeWitt detectors carried by them behave differently. Still both such observers agree that the state they measure is non-vacuum.

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